Spatial Portfolio Theory

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Background (1)

- Classical Theory: Transportation improvement helps increase housing price.

- We contended by a novel hypothesis: Transportation improvement helps flatten housing price gradient; absolute price change is unknown.

- Alonso's (1964) mono-centric bid rent curve.
A Counter-intuitive Hypothesis

Increase Demand
Move in from
External

Decrease Demand
– Move
from A
to B

Higher Demand

NODE A

Lower Demand

NODE B

Road

Bridge

Road

NODE B

NODE A
Old Theory Taken for Granted

– Transportation improvements may decrease housing price?
– People and companies may move from the originally more convenient location to the now equally convenient location due to the completion of the infrastructure.
– i.e. the price gradient between the nodes would be flattened,
– but the change of the absolute magnitude in price cannot be predicted, depending on the net increase in demand.
Developed a Price Gradient Model

• 2 empirical tests on the transportation improvement effects on housing price gradient, rather than on price;
  – Developed a novel empirical regression model with interactive terms to identify the gradient changes of housing price.

• Details see
2 Micro Empirical Evidence

\[ \ln(\text{PRICE }) = \alpha_1 + \alpha_2 AGE + \alpha_3 GFA + \alpha_4 FLR + \alpha_5 SV + \beta_1 TY \times BEF + \beta_2 TY \times AFT + \beta_3 ST \times AFT + \varepsilon \]

- Housing transaction data: 14,767 transactions
Background (2)

• Classical Theory: one-workplace model - traveling frequency determines housing location choice.

• But how about two-workplace?
  – 2 workplaces for husband and wife;
  – cross-border HQs and factory;

• Developed a two-workplace (with known traveling frequency) model:

  • the housing **price gradient** between two workplaces will become flattened when the visiting frequency to the workplace increases relatively.

Two Workplaces (wKF) Model

Fig. 1. The change in price gradient when \( f_2 \) increases and \( f_1 > f_2 \) at time \( t=0 \).
A Novel Temporal-Spatial Differencing Empirical Model

Micro-economics

\[
\ln(P_i) = \beta_0 + \beta_1 \text{AGE}_i + \beta_2 \text{AGE}^2_i + \beta_3 \text{FLR}_i + \beta_4 \text{FLR}^2_i + \beta_5 \text{GFA}_i + \beta_6 \text{GFA}^2_i \\
+ \sum_{t=1}^{\Gamma} \phi_t S_t M_{i,t} + \sum_{t=1}^{\Gamma} \eta_t N_t M_{i,t} + \varepsilon_i
\]  

\[
G_t = \frac{\exp(\phi_t)}{\exp(\eta_t)}
\]

**Housing Price Gradient between the 2 terminals, S & N**

\[
\ln G_t = \ln \text{P}_{A,t} - \ln \text{P}_{B,t} = \beta (\ln S_{A,t} - \ln S_{B,t}) + \gamma (\ln D_{A,t} - \ln D_{B,t}) + \lambda (\ln F_{A,t} - \ln F_{B,t}) \\
+ \theta (\ln Q_{A,t} - \ln Q_{B,t}) + \varphi (\ln C_{A,t} - \ln C_{B,t}) + \ldots
\]

**Differences in the Traveling Frequencies between S & N**

Macro-economics
A Macro Empirical Evidence
HK-Macau

• The 1st paper on Macau housing market;
• better data controls on cross-border proxies;
Motivations

• Classical models are uncertainty-free;
• Previous studies on uncertain transportation cost, uncertain housing price, etc.
• How about two-workplaces, with uncertain traveling frequency?
  – **A new housing centre can be emerged!**
Why Multiple Workplaces?

• Globalisation Trend;
• Contemporary office strategies:
  – Multi-site
  – Shared space
  – Hotel space
  – Just-in-time offices
  – Free address model
• cater for more than one workplace with uncertain traveling frequency!

Theoretical Framework

This paper develops
a two-centre two-workplace with stochastic (uncertain) traveling frequency model
i.e. a (2-2-0) model of a linear bi-centric city of two workplaces

\[ D_1 \quad D \quad D_2 \]

\[ (0,0) \]
Expected Commuting Cost

The expected commuting distance $E(d)$ at any point $(z,0)$ weighted with the frequency, $u$

$$E(d) = \mu_1 z + \mu_2 (D - z)$$

Assuming fixed unit cost of commuting $c$, the expected commuting cost $E(c)$ at any point $(z,0)$ is then:

$$E(c) = c \left[ \mu_1 z + \mu_2 (D - z) \right]$$
Variance of $E(c)$

The variance of the expected commuting cost is:

$$\sigma_c^2 = \text{Var}(c \cdot d)$$

$$= c^2 \text{Var}(d)$$

$$= c^2 \cdot [z^2 \sigma_1^2 + (D - z)^2 \sigma_2^2 + 2 \text{Corr}(D_1, D_2) \cdot \sigma_1 \sigma_2 z(D - z)].$$

Since $\text{Corr}(D_1, D_2) = -1$

$$\sigma_c^2 = c^2 \left[ z^2 \sigma_1^2 + (D - z)^2 \sigma_2^2 - 2Dz \sigma_1 \sigma_2 + 2z^2 \sigma_1 \sigma_2 \right]$$

$$= c^2 \left[ z^2 (\sigma_1^2 + \sigma_2^2 + 2 \sigma_1 \sigma_2) + D^2 \sigma_2^2 - 2Dz \sigma_2^2 - 2Dz \sigma_1 \sigma_2 \right]$$

$$= c^2 \left[ z^2 (\sigma_1 + \sigma_2)^2 + D \sigma_2 (D \sigma_2 - 2z \sigma_2 - 2z \sigma_1) \right]$$
Household's Utility Maximization

\[
\max_{y,h,r} U(x, h, \sigma_c^2) \\
\text{s.t. } Y = x + hR(z) + cE(d) \text{ and } \sigma_c^2 = \sigma_c^2(z)
\]

where
\(x\) is the non-housing composite goods with price equal to one;
\(h\) is the quantity of homogeneous housing consumption.
The household’s utility increases with \(x\), \(h\) and decreases with \(\sigma_c^2\), i.e. \(Ux, Uh > 0\) and \(U\sigma^2c < 0\);
subscripts are the corresponding partial derivatives.
\(Y\) is the household’s income per period,
\(R(.)\) is the function of unit housing rent, and
\(c\) is the fixed unit commuting cost.
Lagrangian of the Maximization

\[ U(x, h, \sigma^2_c) + \lambda [Y - x - hR(z) - cE(d)]. \]

The first order conditions are:

\[ U_x - \lambda^* = 0, \]
\[ U_h - \lambda^* R(z) = 0, \]
\[ U_\sigma \left( \frac{\partial \sigma^2_c}{\partial z} \right) - \lambda^* hR'(z) - \lambda^* c(\mu_1 - \mu_2) = 0, \]

which implies that:

\[ R'(z) = \left[ U_\sigma \left( \frac{\partial \sigma^2_c}{\partial z} \right) - U_x c(\mu_1 - \mu_2) \right] \frac{1}{h \cdot U_x}, \]
\[ R'(z) = -\frac{c(\mu_1 - \mu_2)}{h} + \frac{U_\sigma}{h \cdot U_x} \left( \frac{\partial \sigma^2_c}{\partial z} \right). \]
Differentiate the variance wrt $z$

\[
\frac{\partial \sigma_c^2}{\partial z} = c^2 \left[ 2z \left( \sigma_1^2 + \sigma_2^2 + 2\sigma_1 \sigma_2 \right) - 2D \left( \sigma_a \sigma_b + \sigma_2^2 \right) \right]
\]

Consider the sign of the terms inside the bracket, then

\[
\left[ 2z \left( \sigma_1^2 + \sigma_2^2 + 2\sigma_1 \sigma_2 \right) - 2D \left( \sigma_a \sigma_b + \sigma_2^2 \right) \right] \geq 0
\]

\[
\Rightarrow \frac{z}{D} \geq \frac{\sigma_2^2 + \sigma_1 \sigma_2}{\sigma_1^2 + \sigma_2^2 + 2\sigma_1 \sigma_2} = \theta
\]
A Simple Case

a simple case of \( \sigma_1^2 = \sigma_2^2 \)

\[ \theta = \frac{1}{2} \]

\[ \frac{\partial \sigma_c^2}{\partial z} = c^2 \left[ 2z\left( \sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2 \right) - 2D\left( \sigma_a\sigma_b + \sigma_2^2 \right) \right] = 0 \]

when \( \frac{z}{D} \) \[
\begin{cases}
> 1/2 \\
= 1/2 \\
< 1/2
\end{cases}
\]
Intuitive Interpretations

\[ R'(z) = -\frac{c(\mu_1 - \mu_2)}{h} + \frac{U_\sigma}{h \cdot U_x} \left( \frac{\partial^2 \sigma_c}{\partial z} \right). \]

1. If the traveling frequency is certain, i.e., the 2nd term = 0,
2. then the 1st term tells that the unit housing rent is decreasing with the commuting distance;
3. i.e., Alonosob's bid rent curve.
4. But when traveling frequency is taken into consideration;
5. the sign of the rent gradient is NOT necessarily negative;
6. There are 3 possible cases.
Intuitive Interpretations (Cont'd)

\[ R'(z) = - \frac{c(\mu_1 - \mu_2)}{h} + \frac{U_\sigma}{h \cdot U_x} \left( \frac{\partial \sigma^2_c}{\partial z} \right). \]

1. Even if the expected traveling frequency to centre D1 is higher than that of centre D2;
2. If the commuter dislikes any losses due to the uncertain frequency traveling to centre D2;
3. then the sum of the two terms become positive;
4. i.e the commuter would prefer living closer to the mid point rather than closer to centre D1.
Intuitive Interpretations (Cont'd)

A New Centre can be Emerged between the Two Original Centres!
Empirical Evidence
HK Households work in China within a year
Data and Methodology

transaction data of housing units along the East Rail over the last decades, from 1991 to 2008.
Using standard BMN repeat-sales model
Empirical Model - Repeat Sales

control the original price differences among stations

\[
\ln\left(\frac{P_{it_2}}{P_{it_1}}\right) = \sum_{i=1}^{T} (b_i x_{it} + \sum_{l=1}^{7} S_i \lambda_{lt} x_{it}) + u_{it_1t_2}
\]

where

- \(P\) is transaction price of dwelling,
- \(b\) is the base index,
- \(x\) is equal to 1 when \(t = t_2\) and is equal to \(-1\) when \(t = t_1\),
- \(\lambda\) is the logarithms of the location factor of location,
- \(S = 1\) if dwelling is located at location \(l\); 0 otherwise,
- \(l = 1\) for Tai Wai station, 2 for Sha Tin station… 7 for Sheung Shui station,
- \(T = \) the total number of months in the sample period (1991 - 2008).
- \(\mu\) is the stochastic term.
Empirical Findings (SZ)

• the housing price in the middle part of the East Rail has been increasing relative to other terminus.
Empirical Findings (HK)
Price Contour of Different Locations
Implications

• A spatial portfolio theory in analogous with the portfolio theory in finance.
• Living in the mid point (choosing the ease of travel to the two workplaces in a portfolio) of two workplaces of uncertain traveling frequency acts the role of hedging the risk of changing traveling frequency.
• A new city centre can be emerged between 2 frequently travel workplaces.
Predictions

• The Spatial Portfolio Theory predicts:
  – A new centre at the border between HK and Shenzhen. (a new city is to be built)
  – A new centre to be emerged at Tung Chung when the HK-ZH-MA link bridge is built.
  – The housing price gradient between HK and Macau would be flattened.

• How about the Express Rail?
• Would Chongquin become a new centre for frequent travelers between HK and Shanghai?
The End

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